Available online at www.sciencedirect.com

ScienceDirect

Structural Integrity Procedia 00 (2019) 000–000

Structural Integrity Procedia

www.elsevier.com/locate/procedia

IGF26 - 26th International Conference on Fracture and Structural Integrity

The dynamical problem for the infinite elastic layer with a cylindrical cavity

Anna Fesenko*, Natalya Vaysfel'd

Faculty of Mathematics, Physics and Information Technologies,
Odessa I. I. Mechnikov National University, Dvoryanskaya str., 2, Odessa, 65082, Ukraine

Abstract

The wave field of an infinite elastic layer weakened by a cylindrical cavity is constructed. The ideal contact conditions are given on the upper layer's face and bottom face is rigidly fixed. The normal dynamic tensile load is applied to a cylindrical cavity's surface at the initial moment of time. The Laplace and finite sin- and cos- Fourier integral transforms are applied successively directly to axisymmetric equations of motion and to the boundary conditions, on the contrary to the traditional approaches, when integral transforms are applied to solutions' representation through harmonic and biharmonic functions. This operation leads to a one-dimensional vector inhomogeneous boundary value problem with respect to unknown displacements' transformations. The problem is solved using a matrix differential calculus, which leads to an integral equation solved with a method of orthogonal polynomials. The field of initial displacements is derived after application of inverse integral transforms. The case of the steady-state oscillations was investigated. The normal stress on the rigidly fixed face of the elastic layer is constructed and investigated depending on the mechanical and dynamic parameters. Formulas for determining the normal stress for large values of natural vibration frequencies were constructed.

Keywords: elastic layer; dynamic load; cylindrical cavity; integral transform; integral equation

* Corresponding author. Tel.: +380991457111;
E-mail address: fesenko@onu.edu.ua

1. Introduction

The presence of defects such as inclusions, cracks, cavities in elastic bodies causes a stress concentration and significantly affects at the stress state of constructions. A typical and sufficiently investigated problem of this class is the axisymmetric elasticity problem the stress state of a layer, weakened by a cylindrical defect, when different boundary conditions are set on layer's faces and defect's surface. Existing research can be divided into three approaches: 1) a construction of an analytic solution of the problem in an explicit form (Popov (2013), Grinchenko, Ulitko, (1968), Menshykov et al. (2017)); 2) a construction of an analytical-numerical solution, when the problem is reduced either to an integral equation or to an infinite system of algebraic equations (Malitz, Privarnikov (1971), Arutunyan, Abramyan (1969)); 3) a numerical solving of the problem (Yahnioglu, Babuscu Yesil (2009), Jain, Mittal (2008), Folias, Wang (1990).

For realization of the first approach, it is essential to satisfy the conditions of ideal contact, when the normal displacements and tangential stress are equal to zero. The exact solution of the formulated problem for the case, when the layer is replaced by a half-space and the stresses are given on the faces, is derived by Arutunyan, Abramyan (1969). An approximate analytical - numerical solutions for other boundary conditions on the defect's surface were obtained by Guz' (1962) and Bobyleva (2016).

An exact solution was obtained for an infinite plate with an elliptical hole by Muskhelishvili (1963), Timoshenko and Goodier (1970).

The problem for a plate of finite thickness containing an elliptical hole subjected to a uniaxial tensile stress, using the finite element method considered by Zheng Yang (2009). The relation between stress and strain concentration factors was obtained. The effects of the shape factor of the elliptical hole and the plate thickness on the locations of the maximum stress concentration factor and the strain concentration factor were examined.

Dynamic statement of the mentioned problem was considered by Vorovich and Babeshko (1979), Bardzokas et al. (2009). The theory of harmonic oscillations and wave propagation in elastic bodies was widely investigated in the monograph by Grinchenko and Meleshko (1981). The papers of Kubenko (1965) and Panasyuk (1978) are devoted to the propagation of elastic waves in plates weakened by the cavities or holes. Based on complex function theory, an analytical solution for the dynamic stress concentration due to an arbitrary cylindrical cavity in an infinite inhomogeneous medium was investigated by Baoping Hei et al. (2016). The existence of trapped elastic waves above a circular cylindrical cavity in a half-space was demonstrated by Linton and Thompson (2018).

It should be noted that dynamical problems weakened by the defects have found wide application in the practical problems Zhou et al. (2011), Zhuk et al. (2012). An experimental method was proposed to explore dynamic failure process of pre-stressed rock specimen with a circular hole to investigate deep underground rock failure by Ming Tao et al (2017). A set of exact solutions for three-dimensional dynamic responses of a cylindrical lined tunnel in saturated soil due to internal blast loading are derived by using Fourier transform and Laplace transform proposed by Gaoa et al (2016). The surrounding soil was modeled as a saturated medium on the basis of Biot's theory and the lining structure modeled as an elastic medium. By utilizing a reliable and efficient numerical method of inverse Laplace transform and Fourier transform, the numerical solutions for the dynamic response of the lining and surrounding soil were obtained.

Nevertheless, the study of an elastic layer hasn't been completed yet and many problems are still opened. The main difficulty during the solving of the dynamic problems by the method of integral transforms remains the inversion problem of the Laplace transform. Therefore, it is often necessary to proceed to a more narrow class of the problems about steady state oscillations. Research contributions over the past 50 years on the theory and analysis of elastodynamics are reviewed by Yih-Hsing Pao (1983). Major topics reviewed are: general theories, steady-state waves in waveguides, transient waves in layered media, diffraction and scattering, and one and two-dimensional theories of elastic bodies. A brief discussion on the direct and inverse problems of elastic waves completes this review.

The problem of elasticity for an infinite layer with a cylindrical cavity in a static statement was considered by Popov (2013), where an exact solution was obtained.

In this paper the approach was extended on the analogical problem in the dynamic statement. The matrix differential calculus was used during the solution construction.

2. Statement of the problem.

An elastic layer of thickness h (G is a shear modulus, μ is a Poisson's ratio, ρ is density), describing in the cylindrical coordinate system by the correspondences: $a < r < \infty$, $-\pi < \varphi \le \pi$, $0 \le z \le h$ is weakened by a cylindrical cavity $0 \le r \le a$, $-\pi < \varphi \le \pi$, $0 \le z \le h$ (Fig. 1).

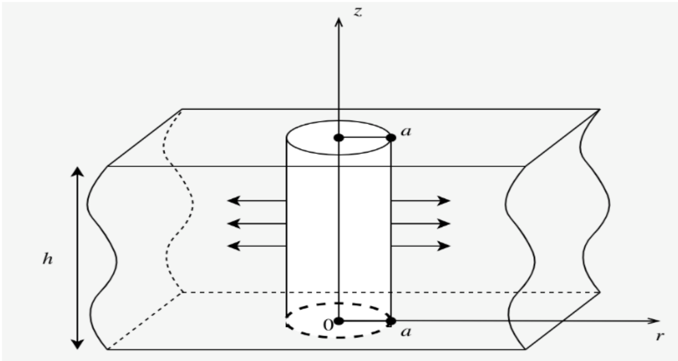


Fig. 1. Geometry of the problem

The layer's upper face $z = h$ is in the conditions of ideal contact with a rigid base (the layer is supported by a smooth foundation without a friction) and the bottom face $z = 0$ is rigidly fixed

$$u_r(r,0,t) = 0,\ u_z(r,0,t) = 0,\ u_z(r,h,t) = 0,\ \tau_{zr}(r,h,t) = 0 \tag{1}$$

The cylindrical cavity's surface $r = a$ is under the influence of the normal dynamic tensile force $P = P(z,t)$, applied at the initial moment $t = 0$, the tangential loading is absent

$$\sigma_r(a,z,t) = P(z,t),\ \tau_{rz}(a,z,t) = 0 \tag{2}$$

Thus, the problem was reduced to solving axisymmetric equations of motion with respect to the functions $u_r(r,z,t) = u(r,z,t)$, $u_z(r,z,t) = w(r,z,t)$ in a cylindrical coordinate system (Novazkiy, 1975)

$$r^{-1}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}u(r,z,t)\right] - r^{-2}u(r,z,t) + \tfrac{\kappa-1}{\kappa+1}\frac{\partial^2}{\partial z^2}u(r,z,t) + \tfrac{2}{\kappa+1}\frac{\partial^2}{\partial r\partial z}w(r,z,t) = \tfrac{\kappa-1}{\kappa+1}\tfrac{\rho}{G}\frac{\partial^2 u(r,z,t)}{\partial t^2}$$
$$r^{-1}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}w(r,z,t)\right] + \tfrac{\kappa+1}{\kappa-1}\frac{\partial^2}{\partial z^2}w(r,z,t) + \tfrac{2}{\kappa-1}r^{-1}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial z}u(r,z,t)\right] = \tfrac{\rho}{G}\frac{\partial^2 w(r,z,t)}{\partial t^2} \tag{3}$$

where $\kappa = 3 - 4\mu$ and subjected to the mixed boundary conditions (1), (2). Here $c_1^2 = \tfrac{\kappa+1}{\kappa-1}G/\rho$ − squared velocity of longitudinal wave propagation, $c^2 = G/\rho$ − squared velocity of shear wave propagation. So, $c_1^2 = \tfrac{\kappa+1}{\kappa-1}c^2$.

The following change of the variables was done

$$\rho = a^{-1}r,\ \xi = h^{-1}z,\ \tau = ca^{-1}t,\ u(a\rho, h\xi, c^{-1}a\tau) = U(\rho,\xi,\tau),\ w(a\rho, h\xi, c^{-1}a\tau) = W(\rho,\xi,\tau) \tag{4}$$

Consequently, the motion equations (3) can be written in the form

$$\rho^{-1}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}U(\rho,\xi,\tau)\right]-\rho^{-2}U(\rho,\xi,\tau)+\tfrac{\kappa-1}{\kappa+1}\alpha^2\frac{\partial^2}{\partial\xi^2}U(\rho,\xi,\tau)+\tfrac{2}{\kappa+1}\alpha\frac{\partial^2}{\partial\rho\partial\xi}W(\rho,\xi,\tau)=\tfrac{\kappa-1}{\kappa+1}\frac{\partial^2 U(\rho,\xi,\tau)}{\partial\tau^2}$$

$$\rho^{-1}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}W(\rho,\xi,\tau)\right]+\tfrac{\kappa+1}{\kappa-1}\alpha^2\frac{\partial^2}{\partial\xi^2}W(\rho,\xi,\tau)+\rho^{-1}\tfrac{2}{\kappa-1}\alpha\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\xi}U(\rho,\xi,\tau)\right]=\frac{\partial^2 W(\rho,\xi,\tau)}{\partial\tau^2} \quad (5)$$

$$1<\rho<\infty,\ 0<\xi<1,\ \alpha=a/h.$$

Boundary conditions (1), taking into account the replacement (4), are transformed into form

$$U(\rho,0,\tau)=0,\ \frac{\partial}{\partial\xi}U(\rho,1,\tau)=0,\ W(\rho,0,\tau)=0,\ \ W(\rho,1,\tau)=0 \quad (6)$$

as the boundary conditions (2) take the form

$$\frac{\partial}{\partial\rho}U(1,\xi,\tau)+\tfrac{3-\kappa}{1+\kappa}\left[U(1,\xi,\tau)+\alpha\frac{\partial}{\partial\xi}W(1,\xi,\tau)\right]=aG^{-1}\tfrac{\kappa-1}{\kappa+1}P(\xi,\tau) \quad (7)$$

$$\alpha\frac{\partial}{\partial\xi}U(1,\xi,\tau)+\frac{\partial}{\partial\rho}W(1,\xi,\tau)=0 \quad (8)$$

3. Solving the vector boundary problem in transform domain.

In order to reduce the problem to the one-dimensional problem, the finite *sin*- and *cos*- Fourier integral transforms with regard of the variable ξ and Laplace integral transform with regard of the variable τ (Sneddon, 1955) are applied successively to the differential equations (5) and boundary conditions (6)-(8)

$$\begin{bmatrix}U_\lambda(\rho,\tau)\\W_\lambda(\rho,\tau)\end{bmatrix}=\int_0^1\begin{bmatrix}U(\rho,\xi,\tau)\cos\lambda_n\xi\\W(\rho,\xi,\tau)\sin\lambda_n\xi\end{bmatrix}d\xi,\quad \begin{matrix}n=0,1,2....\\n=1,2,...\end{matrix}\quad \lambda_n=\pi n=\lambda$$

$$\begin{bmatrix}U_{\lambda p}(\tau)\\W_{\lambda p}(\tau)\end{bmatrix}=\int_0^\infty\begin{bmatrix}U_\lambda(\rho,\tau)\\W_\lambda(\rho,\tau)\end{bmatrix}e^{-p\tau}d\tau$$

As a result, equations (5) can be written

$$\rho^{-1}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}U_{\lambda p}(\rho)\right]+\tfrac{2}{\kappa+1}\lambda_*\frac{\partial}{\partial\rho}W_{\lambda p}(\rho)-\rho^{-2}U_{\lambda p}(\rho)-\tfrac{\kappa-1}{\kappa+1}\lambda_*^2U_{\lambda p}(\rho)-\tfrac{\kappa-1}{\kappa+1}p^2U_{\lambda p}(\rho)=\chi(\rho)$$

$$\rho^{-1}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}W_{\lambda p}(\rho)\right]-\rho^{-1}\tfrac{2}{\kappa-1}\lambda_*\frac{\partial}{\partial\rho}\left[\rho U_{\lambda p}(\rho)\right]-\tfrac{\kappa+1}{\kappa-1}\lambda_*^2W_{\lambda p}(\rho)-p^2W_{\lambda p}(\rho)=0 \quad (9)$$

$$\chi(\rho)=\frac{\partial}{\partial\xi}U_p(\rho,0),\ \ \lambda_*=\lambda\alpha\ ,\ 1<\rho<\infty$$

During this operation the boundary conditions (6) are automatically satisfied except the first condition $U(\rho,0,\tau)=0$, which derivative goes to the right part of the equations (9). Boundary conditions (7), (8) take the form

$$U'_{\lambda p}(1)+\tfrac{3-\kappa}{1+\kappa}\left[U_{\lambda p}(1)+\lambda_* W_{\lambda p}(1)\right]=aG^{-1}\tfrac{\kappa-1}{\kappa+1}p_{\lambda p}\,,\quad W'_{\lambda p}(1)-\lambda_* U_{\lambda p}(1)=0\,,$$

$$p_{\lambda p}=\int\limits_0^{\infty}\left(\int\limits_0^1 P(\xi,\tau)\cos\lambda_n\xi d\xi\right)e^{-p\tau}d\tau \tag{10}$$

For solving a one-dimensional boundary value problem (9), (10) a second-order matrix differential operator and the unknown vector of displacements' transformations are set

$$L_2=\begin{pmatrix}\rho^{-1}\dfrac{\partial}{\partial\rho}\left[\rho\dfrac{\partial}{\partial\rho}\right]-\rho^{-2}-\tfrac{\kappa-1}{\kappa+1}\left(\lambda_*^2+p^2\right) & \tfrac{2}{\kappa+1}\lambda_*\dfrac{\partial}{\partial\rho}\\ -\tfrac{2}{\kappa-1}\lambda_*\rho^{-1}\dfrac{\partial}{\partial\rho}[\rho] & \rho^{-1}\dfrac{\partial}{\partial\rho}\left[\rho\dfrac{\partial}{\partial\rho}\right]-\tfrac{\kappa+1}{\kappa-1}\lambda_*^2-p^2\end{pmatrix},\qquad \mathbf{y}(\rho)=\begin{pmatrix}U_{\lambda p}(\rho)\\ W_{\lambda p}(\rho)\end{pmatrix}$$

The boundary functional corresponding to the boundary conditions (10) is written in the form

$$\mathrm{U}\left[\mathbf{y}(1)\right]=\mathbf{A}\cdot\mathbf{y}(1)+\mathbf{I}\cdot\mathbf{y}'(1)\,,\ \mathbf{A}=\begin{pmatrix}\tfrac{3-\kappa}{1+\kappa} & \tfrac{3-\kappa}{1+\kappa}\lambda_*\\ -\lambda_* & 0\end{pmatrix},\ \mathbf{I}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

In these notations the boundary value problem (9), (10) is derived

$$L_2\mathbf{y}(\rho)=\mathbf{f}(\rho),\ 1<\rho<\infty,\quad \mathrm{U}\left[\mathbf{y}(1)\right]=\boldsymbol{\gamma} \tag{11}$$

$$\mathbf{f}(\rho)=\begin{pmatrix}\chi(\rho)\\ 0\end{pmatrix},\ \boldsymbol{\gamma}=\begin{pmatrix}aG^{-1}\tfrac{\kappa-1}{\kappa+1}p_{\lambda p}\\ 0\end{pmatrix}$$

A general solution of the vector homogeneous equation in (11) relates to the solution of the matrix differential equation

$$L_2\mathbf{Y}(\rho)=0,\ 1<\rho<\infty \tag{12}$$

with the help of the auxiliary matrix

$$\mathbf{H}(\rho,\xi)=\begin{pmatrix}H_1^{(1)}(\rho\xi) & 0\\ 0 & H_0^{(1)}(\rho\xi)\end{pmatrix}$$

where $H_m^{(1)}(z)$ is the Hankel first order function, $m=0,1$, a relationship has been proven (Popov, 2013)

$$L_2\mathbf{H}(\rho,\xi)=-\mathbf{H}(\rho,\xi)\cdot\mathbf{M}(\xi)\,,\ \mathbf{M}(\xi)=\begin{pmatrix}\xi^2+\tfrac{\kappa-1}{\kappa+1}\left(\lambda_*^2+p^2\right) & \tfrac{2}{\kappa+1}\xi\lambda_*\\ \tfrac{2}{\kappa-1}\xi\lambda_* & \xi^2+\tfrac{\kappa+1}{\kappa-1}\lambda_*^2+p^2\end{pmatrix} \tag{13}$$

The inverse matrix for $\mathbf{M}(\xi)$ has the form

$$\mathbf{M}^{-1}(\xi)=\frac{1}{\det\mathbf{M}}\begin{pmatrix}\xi^2+\frac{\kappa+1}{\kappa-1}\lambda_*^2+p^2 & -\frac{2}{\kappa+1}\xi\lambda_* \\ -\frac{2}{\kappa-1}\xi\lambda_* & \xi^2+\frac{\kappa-1}{\kappa+1}\left(\lambda_*^2+p^2\right)\end{pmatrix}$$

$$\det\mathbf{M}=\left[\xi-i\sqrt{\lambda_*^2+p^2}\right]\left[\xi+i\sqrt{\lambda_*^2+p^2}\right]\left[\xi-i\sqrt{\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2}\right]\left[\xi+i\sqrt{\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2}\right]$$

Further, with the help of the equality (13), one can be convinced that the solution of the matrix equation (12) is

$$\mathbf{Y}(\rho)=\frac{1}{2\pi i}\int_C \mathbf{H}(\rho,\xi)\cdot\mathbf{M}^{-1}(\xi)d\xi$$

where C is the closed loop covering the origin and two poles of the first multiplicity $\xi=i\sqrt{\lambda_*^2+p^2}$, $\xi=i\sqrt{\lambda_*^2+\frac{\kappa-1}{\kappa+1}p^2}$ lying in the upper half-plane. Applying the methods of contour integration, the matrix is derived using the residue theorem

$$\mathbf{Y}(\rho)=\frac{1}{2p^2}\begin{pmatrix} i\frac{\kappa+1}{\kappa-1}\frac{\lambda_*^2}{\delta_1}H_1^{(1)}\left(i\rho\delta_1\right) & \lambda_* H_1^{(1)}\left(i\rho\delta_1\right) \\ \frac{\kappa+1}{\kappa-1}\lambda_* H_0^{(1)}\left(i\rho\delta_1\right) & -i\delta_1 H_0^{(1)}\left(i\rho\delta_1\right)\end{pmatrix}+\frac{1}{2p^2}\begin{pmatrix} -i\frac{\kappa+1}{\kappa-1}\delta_2 H_1^{(1)}\left(i\rho\delta_2\right) & -\lambda_* H_1^{(1)}\left(i\rho\delta_2\right) \\ -\frac{\kappa+1}{\kappa-1}\lambda_* H_0^{(1)}\left(i\rho\delta_2\right) & i\frac{\lambda_*^2}{\delta_2}H_0^{(1)}\left(i\rho\delta_2\right)\end{pmatrix}$$

where

$$\delta_1=\sqrt{\lambda_*^2+p^2},\ \ \delta_2=\sqrt{\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2}$$

Taking real and imaginary part of the matrix $\mathbf{Y}(\rho)$, increasing and decreasing, when $\rho\to\infty$, solutions of the homogeneous matrix equation (12) are constructed

$$\mathbf{Y}_{\lambda p}^{R}(\rho)=\frac{1}{p^2}\begin{pmatrix} -\frac{\kappa+1}{\kappa-1}\frac{\lambda_*^2}{\delta_1}I_1\left(\rho\delta_1\right) & \lambda_* I_1\left(\rho\delta_1\right) \\ \frac{\kappa+1}{\kappa-1}\lambda_* I_0\left(\rho\delta_1\right) & -\sqrt{\lambda_*^2+p^2}\,I_0\left(\rho\delta_1\right)\end{pmatrix}+\frac{1}{p^2}\begin{pmatrix} \frac{\kappa+1}{\kappa-1}\delta_2 I_1\left(\rho\delta_2\right) & -\lambda_* I_1\left(\rho\delta_2\right) \\ -\frac{\kappa+1}{\kappa-1}\lambda_* I_0\left(\rho\delta_2\right) & \frac{\lambda_*^2}{\delta_2}I_0\left(\rho\delta_2\right)\end{pmatrix}$$

$$\mathbf{Y}_{\lambda p}^{S}(\rho)=\frac{1}{p^2}\begin{pmatrix} -\frac{\kappa+1}{\kappa-1}\frac{\lambda_*^2}{\delta_1}K_1\left(\rho\delta_1\right) & -\lambda_* K_1\left(\rho\delta_1\right) \\ \frac{\kappa+1}{\kappa-1}\lambda_* K_0\left(\rho\delta_1\right) & -\sqrt{\lambda_*^2+p^2}\,K_0\left(\rho\delta_1\right)\end{pmatrix}+\frac{1}{p^2}\begin{pmatrix} \frac{\kappa+1}{\kappa-1}\delta_2 K_1\left(\rho\delta_2\right) & \lambda_* K_1\left(\rho\delta_2\right) \\ -\frac{\kappa+1}{\kappa-1}\lambda_* K_0\left(\rho\delta_2\right) & \frac{\lambda_*^2}{\delta_2}K_0\left(\rho\delta_2\right)\end{pmatrix}$$

where $I_m(z)$ are Infeld functions, $K_m(z)$ are the Macdonald functions, $m=0,1$.
Matrices $\mathbf{Y}_{\lambda p}^{S}(\rho)$, $\mathbf{Y}_{\lambda p}^{R}(\rho)$ are singular and regular in zero.

Basis matrix can be constructed in the form (Popov et al., 1999)

$$\mathbf{\Psi}(\rho)=\mathbf{Y}_{\lambda p}^{R}(\rho)\mathbf{C}_0+\mathbf{Y}_{\lambda p}^{S}(\rho)\mathbf{C}_1$$

where matrix coefficients $\mathbf{C}_0$, $\mathbf{C}_1$ are found after satisfying the condition $U\left[\mathbf{\Psi}(\rho)\right]=0$, so

$$\mathbf{\Psi}(\rho)=\mathbf{Y}^{R}_{\lambda p}(\rho)-\mathbf{Y}^{S}_{\lambda p}(\rho)\mathbf{D},\quad \mathbf{D}=\left[\mathbf{A}\cdot\mathbf{Y}^{S}_{\lambda p}(1)+\mathbf{Y}^{S\,\prime}_{\lambda p}(1)\right]^{-1}\left[\mathbf{A}\cdot\mathbf{Y}^{R}_{\lambda p}(1)+\mathbf{Y}^{R\,\prime}_{\lambda p}(1)\right]$$

After calculation, matrix $\mathbf{\Psi}(\rho)$ is found in the form

$$\mathbf{\Psi}(\rho)=\frac{1}{p^2}\begin{pmatrix}\psi_{11}(\rho) & \psi_{12}(\rho)\\ \psi_{21}(\rho) & \psi_{22}(\rho)\end{pmatrix}$$

Components of the matrix are given in the Appendix A.

The solution to the one-dimensional problem (11) is written in the form (Popov et al. 1999)

$$\mathbf{y}(\rho)=\int_1^\infty \mathbf{G}(\rho,r)\mathbf{f}(r)dr+\mathbf{\Psi}(\rho)\boldsymbol{\gamma} \tag{14}$$

$\mathbf{G}(\rho,r)$ is a Green matrix function. Note that the product $\mathbf{\Psi}(\rho)\boldsymbol{\gamma}$ equals to $\mathbf{y}^0(\rho)$, where vector solution $\mathbf{y}^0(\rho)$ relates to the exact solution of the analogical problem for the layer, when the conditions of a smooth contact are set on the bottom layer's face, it was constructed earlier (Fesenko, 2019) and has a form

$$\begin{aligned}U^0_{\lambda p}(\rho)&=\frac{a}{G}p_{\lambda p}\frac{\delta_2}{\Delta}\left[2\lambda_*^2K_1(\rho\delta_1)K_1(\delta_2)-\left(2\lambda_*^2+p^2\right)K_1(\rho\delta_2)K_1(\delta_1)\right]\\ W^0_{\lambda p}(\rho)&=\frac{a}{G}p_{\lambda p}\frac{\lambda_*}{\Delta}\left[2\delta_1\delta_2K_0(\rho\delta_1)K_1(\delta_2)-\left(2\lambda_*^2+p^2\right)K_0(\rho\delta_2)K_1(\delta_1)\right]\end{aligned} \tag{15}$$

where

$$\begin{aligned}\Delta=&\tfrac{\kappa+1}{\kappa-1}\left(\lambda_*^2+\tfrac{1}{2}p^2\right)\left(\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2\right)K_1(\delta_1)K_2(\delta_2)-\tfrac{\kappa+1}{\kappa-1}\lambda_*^2\delta_1\delta_2K_1(\delta_2)K_2(\delta_1)-\\ &-\tfrac{3-\kappa}{\kappa-1}p^2\delta_2K_1(\delta_1)K_1(\delta_2)-\left(\lambda_*^2+\tfrac{1}{2}p^2\right)\left(\tfrac{5-3\kappa}{\kappa-1}\lambda_*^2-p^2\right)K_1(\delta_1)K_0(\delta_2)+\tfrac{5-3\kappa}{\kappa-1}\lambda_*^2\delta_1\delta_2K_1(\delta_2)K_0(\delta_1)\end{aligned} \tag{16}$$

Expression for $p_{\lambda p}$ is given in (10).

4. Construction of the Green matrix function.

The Green matrix function is constructed in the form

$$\mathbf{G}(\rho,r)=\begin{cases}\mathbf{\Psi}(\rho)\mathbf{C}(r), & 1<\rho<r\\ \mathbf{Y}^{S}_{\lambda p}(\rho)\mathbf{B}(r), & r<\rho<\infty\end{cases} \tag{17}$$

To find the unknown matrix $\mathbf{B}(r)$ one must satisfy the continuous conditions

$$\mathbf{\Psi}(\rho)\mathbf{C}(\rho)=\mathbf{Y}^{S}_{\lambda p}(\rho)\mathbf{B}(\rho)\ \Rightarrow\ \mathbf{B}(\rho)=\left[\mathbf{Y}^{S}_{\lambda p}(\rho)\right]^{-1}\mathbf{\Psi}(\rho)\mathbf{C}(\rho)$$

Using the discontinuity property of the prime derivative of Green function

$$\mathbf{G}'(r+0,r)-\mathbf{G}'(r-0,r)=\mathbf{I}$$

matrix $\mathbf{C}(r)$ was found

$$\mathbf{C}(r)=\left[\mathbf{Y}_{\lambda p}^{S}{}'(r)\cdot\left[\mathbf{Y}_{\lambda p}^{S}(r)\right]^{-1}\cdot\mathbf{Y}_{\lambda p}^{R}(r)-\mathbf{Y}_{\lambda p}^{R}{}'(r)\right]^{-1}$$

The element form of Green matrix function (16) is derived

$$\mathbf{G}(\rho,r)=\begin{cases}\begin{pmatrix} g_{11}^{1} & g_{12}^{1} \\ g_{21}^{1} & g_{22}^{1}\end{pmatrix}, & 1<\rho<r \\ \begin{pmatrix} g_{11}^{2} & g_{12}^{2} \\ g_{21}^{2} & g_{22}^{2}\end{pmatrix}, & r<\rho<\infty\end{cases}$$

The solution (14) can be written in a form

$$\begin{aligned}U_{\lambda p}(\rho)&=\int_{1}^{\rho}g_{11}^{2}(\rho,r)\chi(r)dr+\int_{\rho}^{\infty}g_{11}^{1}(\rho,r)\chi(r)dr+U_{\lambda p}^{0}(\rho)\\W_{\lambda p}(\rho)&=\int_{1}^{\rho}g_{21}^{2}(\rho,r)\chi(r)dr+\int_{\rho}^{\infty}g_{21}^{1}(\rho,r)\chi(r)dr+W_{\lambda p}^{0}(\rho)\end{aligned}\tag{18}$$

where functions $g_{11}^{2}(\rho,r),\ g_{11}^{1}(\rho,r),\ g_{21}^{2}(\rho,r),\ g_{21}^{1}(\rho,r)$ are given in the Appendix B.

5. Inverse integral transform and the case of steady oscillations.

After applying to the solution (18) the inverse Laplace and finite cos- , sin- transforms

$$U_p(\rho,\xi)=U_{0p}(\rho)+2\sum_{n=1}^{\infty}U_{\lambda p}(\rho)\cos\left(\lambda_n\xi\right),\ \ W_p(\rho,\xi)=2\sum_{n=1}^{\infty}W_{\lambda p}(\rho)\sin\left(\lambda_n\xi\right),\ \lambda_n=\pi n$$

displacements of the initial problem (5 - 8) take a form

$$\begin{aligned}U\left(\rho,\xi,\tau\right)=\frac{1}{2\pi i}\int_{j-i\infty}^{j+i\infty}\Bigg[&\frac{a}{4G}\rho\int_{0}^{1}p\left(h\xi\right)d\xi+\frac{2a}{G}\sum_{n=1}^{\infty}\cos\left(\lambda_n\xi\right)p_{\lambda p}\frac{\delta_2}{\Delta}F_1\left(\rho\right)+\\&+2\sum_{n=1}^{\infty}\left(\cos\left(\lambda_n\xi\right)\left\{\int_{1}^{\rho}g_{11}^{2}\left(\rho,r\right)\chi(r)dr+\int_{\rho}^{\infty}g_{11}^{1}\left(\rho,r\right)\chi(r)dr\right\}\right)\Bigg]e^{p\tau}dp\end{aligned}\tag{19}$$

$$W(\rho,\xi,\tau)=\frac{1}{2\pi i}\int_{j-i\infty}^{j+i\infty}\left[\frac{2a}{G}\sum_{n=1}^{\infty}\sin(\lambda_n\xi)\,p_{\lambda p}\,\frac{\lambda_*}{\Delta}F_2(\rho)+\right.$$
$$\left.+2\sum_{n=1}^{\infty}\left(\sin(\lambda_n\xi)\left\{\int_1^{\rho}g_{21}^{2}(\rho,r)\chi(r)dr+\int_{\rho}^{\infty}g_{21}^{1}(\rho,r)\chi(r)dr\right\}\right)\right]e^{p\tau}dp$$

The case of steady-state oscillations is considered below. With this aim the substitution $p=i\omega$, $p^2=-\omega^2$ was made (p − Laplace transform parameter, ω − circular frequency of steady-state oscillations).

$$U(\rho,\xi;\omega)=\frac{a}{4G}\rho+\frac{2a}{G}\sum_{n=1}^{\infty}p_{\lambda}\cos(\lambda_n\xi)\frac{\Delta_2}{\det}F_n^{(1)}(\rho;\omega)+$$
$$+2\sum_{n=1}^{\infty}\cos(\lambda_n\xi)\left\{\int_1^{\rho}g_{11}^{2(n)}(\rho,r;\omega)\chi(r)dr+\int_{\rho}^{\infty}g_{11}^{1(n)}(\rho,r;\omega)\chi(r)dr\right\}; \tag{20}$$
$$W(\rho,\xi;\omega)=\frac{2a}{G}\sum_{n=1}^{\infty}p_{\lambda}\sin(\lambda_n\xi)\frac{\lambda_*}{\det}F_n^{(2)}(\rho,\omega)+$$
$$+2\sum_{n=1}^{\infty}\sin(\lambda_n\xi)\left\{\int_1^{\rho}g_{21}^{2(n)}(\rho,r;\omega)\chi(r)dr+\int_{\rho}^{\infty}g_{21}^{1(n)}(\rho,r;\omega)\chi(r)dr\right\};$$

$$\det=-4\lambda_*^{2}\Delta_1\Delta_2K_0(\Delta_1)K_1(\Delta_2)+\left(2\lambda_*^{2}-\omega^2\right)^2K_0(\Delta_2)K_1(\Delta_1)-2\omega^2\Delta_2K_1(\Delta_2)K_1(\Delta_1)$$

$$F_n^{(1)}(\rho;\omega)=2\lambda_*^{2}\mathrm{K}_1(\rho\Delta_1)\mathrm{K}_2(\Delta_2)-\left(2\lambda_*^{2}-\omega^2\right)\mathrm{K}_1(\rho\Delta_2)\mathrm{K}_1(\Delta_1);$$
$$F_n^{(2)}(\rho;\omega)=2\Delta_1\Delta_2K_0(\rho\Delta_1)K_1(\Delta_2)-\left(2\lambda_*^{2}-\omega^2\right)K_0(\rho\Delta_2)K_1(\Delta_1)$$

$$\Delta_1=\sqrt{\lambda_*^{2}-\omega^2};\quad \Delta_2=\sqrt{\lambda_*^{2}-\tfrac{\kappa-1}{\kappa+1}\omega^2};$$

Functions $g_{11}^{i(n)}(\rho,r;\omega)$, $g_{21}^{i(n)}(\rho,r;\omega)$, $i=1,2$ are corresponding to functions $g_{i1}^{i}(\rho,r)$, $i=1,2$, can be found in Appendix B with a replacement $p^2=-\omega^2$.

6. Deriving and solving the integral equation.

Since one boundary condition $U(\rho,0,\tau)=0$ remained unfulfilled, one requires its fulfilment, using expression (19). The integral equation is obtained in the form

$$\sum_{n=1}^{\infty}\int_0^{\infty}F_n(\rho,r;\omega)\chi(r)dr=f(\rho;\omega),\quad 0<\rho<\infty \tag{21}$$

where unknown function $\chi(r)=\frac{\partial}{\partial\xi}U_p(r,0)$ has been extended with zero in the interval $r\in[0,1)$.

$$\int_0^\infty F_n(\rho,r;\omega)\chi(r)dr = \int_1^\rho g_{11}^{2(n)}(\rho,r;\omega)\chi(r)dr + \int_\rho^\infty g_{11}^{1(n)}(\rho,r;\omega)\chi(r)dr;$$

$$f(\rho;\omega) = -\frac{a}{8G}\rho - \frac{a}{G}\sum_{n=1}^{\infty} p_\lambda \frac{\delta_2}{\det} F_n^{(1)}(\rho;\omega)$$

The solution to the integral equation is represented in the series expansion

$$\chi(r) = r^\gamma e^{-r}\sum_{m=0}^{\infty}\chi_m L_m^{(\gamma)}(r) \tag{22}$$

here χ_m - unknown constants, $L_m^{(\gamma)}(r)$ - Laguerre polynomials.

After completing the orthogonalization, i.e. multiplying the equation by $\rho^\gamma e^{-\rho} L_k^{(\gamma)}(\rho)$ and integrating it in the interval $\rho \in [0,\infty)$, the infinite system of algebraic equations of the first kind is derived

$$\sum_{m=0}^{\infty}\chi_m A_{mk} = f_k,\ k = 0,1,2,... \tag{23}$$

where

$$A_{mk} = \sum_{n=1}^{\infty}\int_0^\infty\int_0^\infty F_n(\rho,r;\omega)(r\rho)^\gamma e^{-r-\rho} L_m^{(\gamma)}(r) L_k^{(\gamma)}(\rho)\,dr\,d\rho, \quad f_k = \int_0^\infty f(\rho;\omega)\rho^\gamma e^{-\rho} L_k^{(\gamma)}(\rho)\,d\rho$$

To find the value of γ, the mechanical sense of unknown function $\chi(r)$ should be analysed. Considering the formula of tangential stress $\tau_{rz} = G\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial r}\right)$ and taking into account boundary condition in (6) $W(\rho,0,\tau) = 0$, the relation $\tau_{rz}\big|_{z=0} = G\frac{\partial U}{\partial z}\Big|_{z=0}$ is obtained. Therefore $\frac{\partial}{\partial \xi}U_p(r,0) = \chi(r) \sim \tau_{rz}$.

Analysing the type of singularity that appear to be in the mixed problems for a wedge (Uflyand, 1968), when an angle equals $\pi/2$, it was found that γ depends on Poisson’s ratio μ: for $\mu = 1/4$ $\gamma = 0.2552$; for $\mu = 1/3$ $\gamma = 0.3100$.

After solving the integral equation (21) the unknown function (22) should be substituted into expressions for displacements (20). It leads to the final solution to the given problem (5 - 8) in the case of steady-state oscillations

$$U(\rho,\xi;\omega) = \frac{a}{4G}\rho + \frac{2a}{G}\sum_{n=1}^{\infty} p_\lambda \cos(\lambda_n\xi)\frac{\delta_2}{\det}F_n^{(1)}(\rho;\omega) +$$
$$+2\sum_{n=1}^{\infty}\cos(\lambda_n\xi)\left\{\sum_{m=0}^{N}\chi_m\int_1^\rho g_{11}^{2(n)}(\rho,r;\omega) r^\gamma e^{-r} L_m^{(\gamma)}(r)dr + \sum_{m=0}^{N}\chi_m\int_\rho^\infty g_{11}^{1(n)}(\rho,r;\omega) r^\gamma e^{-r} L_m^{(\gamma)}(r)dr\right\}$$

$$W(\rho,\xi;\omega) = \frac{2a}{G}\sum_{n=1}^{\infty} p_\lambda \sin(\lambda_n\xi)\frac{\lambda_*}{\det}F_n^{(2)}(\rho,\omega) + \tag{24}$$
$$+2\sum_{n=1}^{\infty}\sin(\lambda_n\xi)\left\{\sum_{m=0}^{N}\chi_m\int_1^\rho g_{21}^{2(n)}(\rho,r;\omega) r^\gamma e^{-r} L_m^{(\gamma)}(r)dr + \sum_{m=0}^{N}\chi_m\int_\rho^\infty g_{21}^{1(n)}(\rho,r;\omega) r^\gamma e^{-r} L_m^{(\gamma)}(r)dr\right\}$$

The normal stress of the layer is derived with the help of displacements (24)

$$\sigma_\xi(\rho,\xi;\omega)=\sigma_\xi^0(\rho,\xi;\omega)+ \tag{25}$$
$$+\tfrac{3-\kappa}{\kappa-1}\tfrac{2G}{a}\sum_{n=1}^{\infty}\cos\lambda_n\xi\cdot\left\{\sum_{m=0}^{N}\chi_m\int_1^{\rho}G_2(\rho,r;\omega)r^{\gamma}e^{-r}L_m^{(\gamma)}(r)dr+\sum_{m=0}^{N}\chi_m\int_{\rho}^{\infty}G_1(\rho,r;\omega)r^{\gamma}e^{-r}L_m^{(\gamma)}(r)dr\right\}$$

where

$$G_2(\rho,r;\omega)=\tfrac{\partial}{\partial\rho}g_{11}^{2(n)}+\rho^{-1}g_{11}^{2(n)}+\tfrac{1+\kappa}{3-\kappa}\alpha\lambda_n g_{21}^{2(n)};\; G_1(\rho,r;\omega)=\tfrac{\partial}{\partial\rho}g_{11}^{1(n)}+\rho^{-1}g_{11}^{1(n)}+\tfrac{1+\kappa}{3-\kappa}\alpha\lambda_n g_{21}^{1(n)};$$
$$\tfrac{\partial}{\partial\rho}g_{11}^{1(n)}(\rho,r;\omega)=\frac{2}{\Delta_{C(r)}}\left[c_{11}(r)\psi_{11}'(\rho)+c_{21}(r)\psi_{12}'(\rho)\right];$$
$$\tfrac{\partial}{\partial\rho}g_{11}^{2(n)}(\rho,r;\omega)=\frac{2}{\Delta_{C(r)}\Delta}\Big[\{c_{11}(r)\psi_{11}(r)+c_{12}(r)\psi_{12}(r)\}\{s_{11}'(\rho)r_{22}(r)-s_{12}'(\rho)r_{21}(r)\}+$$
$$+\{c_{11}(r)\psi_{21}(r)+c_{21}(r)\psi_{22}(r)\}\{s_{12}'(\rho)r_{11}(r)-s_{11}'(\rho)r_{12}(r)\}\Big];$$

Expressions for $\psi_{11}'(\rho),\ \psi_{12}'(\rho),\ s_{11}'(\rho),\ s_{12}'(\rho)$ are represented in Appendix C. Here dashes denote the derivative.

The normal stress $\sigma_\xi^0(\rho,\xi;\omega)$ corresponds to the situation when conditions of ideal contact are set on the upper and bottom layer's faces

$$\sigma_\xi^0(\xi,\rho;\omega)=\tfrac{3-\kappa}{2}p_{0\lambda}+4\sum_{n=1}^{\infty}p_{n\lambda}\cos(\alpha\pi n\xi)\frac{\Phi_n(\rho;\omega)}{\det_n} \tag{26}$$

The form of functions $\Phi_n(\rho;\omega)$ and $\det_n$, as well as the method of obtaining them are described in detail in the Appendix D.

7. Discussion and numerical results.

The normal stress (25), (26) on the lower face of the layer $\xi=0$, $1<\rho<\infty$ was investigated, depending on different characteristics: Poisson's ratio $\mu=1/3$ or $\mu=1/4$; ratio of cavity's radius to layer thickness $\alpha=a/h$, $\alpha=1/2$, $\alpha=1/5$; different cases of natural oscillation frequencies $\omega=1;\ 2;\ 5;\ 7$; three types of acting across cavity's surface load

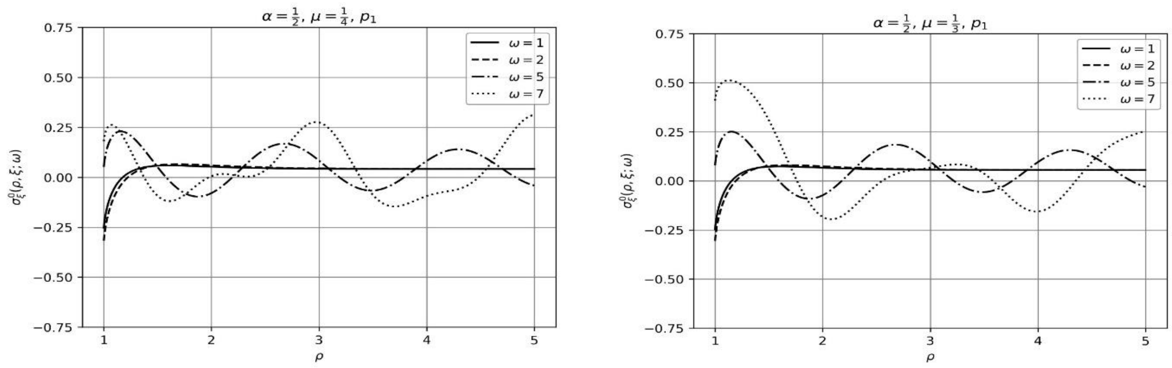
$$p1)\;\; p(\xi)=-\left(\xi-\tfrac{1}{2}\right)^2+\tfrac{1}{4};\quad p2)\;\; p(\xi)=2\left(\xi-\tfrac{1}{2}\right)^2;\quad p3)\;\; p(\xi)=\tfrac{1}{4}\sin(10\xi);$$

The possibility of an appearance of tensile stress on the lower face of the layer was considered. The graphs of normal stress (26) on lower layer's face are presented on Figures 2-4 when both layer's faces are under condition of smooth contact with foundation. Here functions $\Phi_n(\rho;\omega)$ and $\det_n$ were chosen depending on the interval of membership *n*. On Figure 2 comparison of normal stress is carried out depending on Poisson ratio with the same $\alpha=1/2$ and load corresponds to parabola with branches going down ($p1$). Greater stress appear in the material with greater Poisson's ratio. Stress graphs are represented as a wave oscillating near the ρ axes. Visualising zones of stretching stress signalize about lifting the layer's face that lying on the foundation without friction. With the help of

Figures 2a and 3 effect of load can be analysed. Here $\alpha=1/2$ and Poisson ratio $\mu=1/4$. Denote that for $\alpha=1/2$ the necessity of using asymptotic formulas appear from frequency $\omega=2$. Rapid growth of oscillation's amplitude is observed for $\omega=7$ for the load with sine low (*p*3) (Fig. 3b). For frequencies $\omega=1$, $\omega=2$ stress are practically strait line that stabilize near zero and achieve maximum near cavity's surface. Essential wave motion manifests from values of natural frequencies equal three. In general, with an increase in the frequency of oscillation, the amplitude grows.

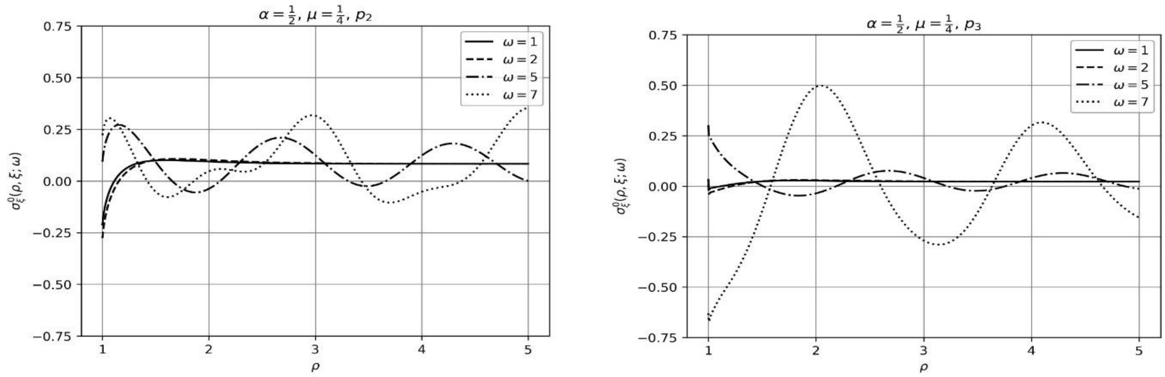
Stress analysis depending on parameter α is given on Figure 4 with the same Poisson ratio and load. Parameter $\alpha=1/2$ on Fig. 4a and $\alpha=1/7$ on Fig. 4b. Decreasing the value of α means that with a constant cavity radius, the layer thickness increases. Amplitude of oscillations decrease when layer thickness increases

Figure 5 represents graphs of normal stress (25) on the rigidly fixed layer's face, when $\alpha=1/2$ and $\alpha=1/5$ respectively, with Poisson ratio $\mu=1/4$ and frequency $\omega=0.3$ for three cases of load that denoted as *p*1, *p*2, *p*3. All values of normal stress here are strictly negative that means that lifting zones of layer's edge are not observed. Maximum absolute values of stress are achieved near cavity's surface and while moving away from cavity stress decrease and stabilize. Minimal values appear when load is parabola with branches up (*p*2) and maximal for sine low (*p*3). Denote that a normal stress on fixed layer face is an approximate solution as it was constructed after solving an integral equation (21). Formula (25) are applicable for small values of frequencies.



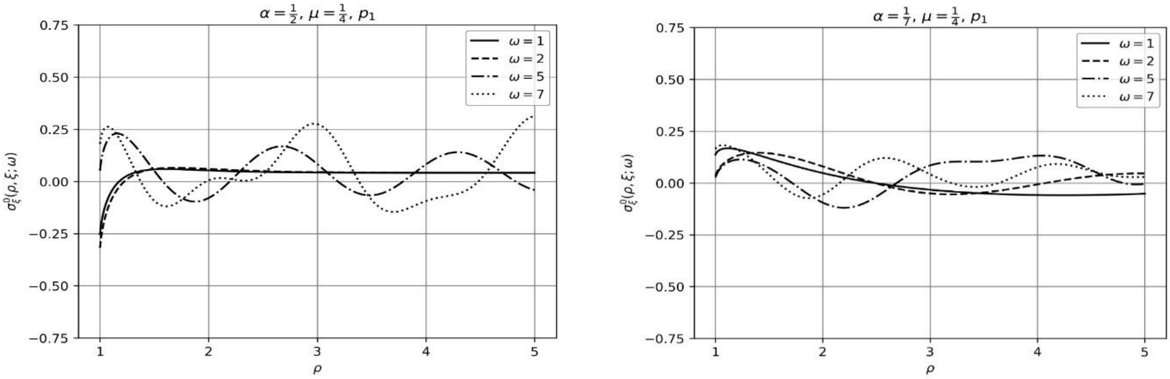
(a) (b)

Fig. 2. Comparison the normal stress depend on Poisson ratio. (a) $\mu=1/4$; (b) $\mu=1/3$;



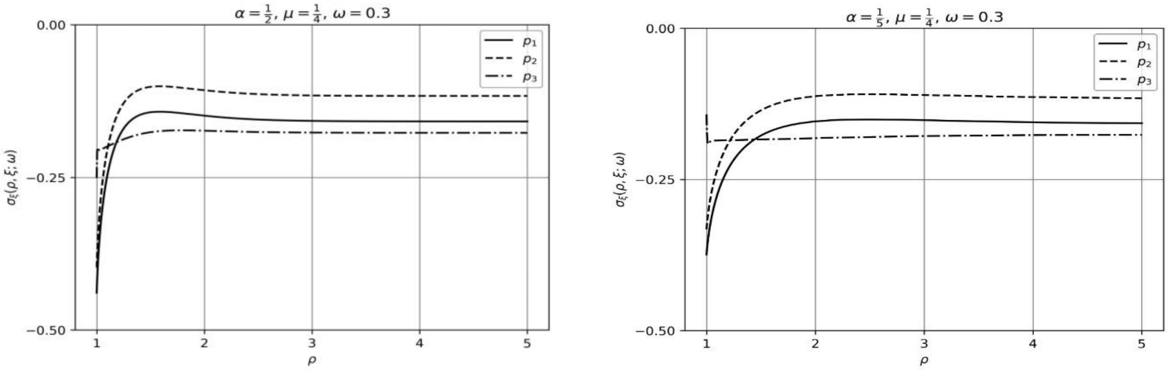
(a) (b)

Fig. 3. Comparison the normal stress depend on acting load. (a) $p2$; (b) $p3$;



(a) (b)

Fig. 4. Comparison the normal stress depend on ratio α . (a) $\alpha = 1/2$; (b) $\alpha = 1/7$;



(a) (b)

Fig. 5. Approximate solution. Comparison the normal stress depend on ratio α . (a) $\alpha = 1/2$; (b) $\alpha = 1/5$;

8. Conclusions.

The dynamic problem's solution of the elasticity for an infinite layer weakened by a cylindrical cavity was derived, when one face of the infinite layer is under the ideal contact conditions and another is rigidly fixed with foundation, dynamic load is applied across the cylindrical cavity's surface.

At the subcase of the ideal contact conditions on both layer's faces, the proposed approach makes it possible to obtain an exact solution of the problem.

When layer's face is rigidly fixed the approximate solution was constructed. Using the method of integral transforms the initial problem was reduced to an inhomogeneous vector differential equation, for its solving Green's matrix function and basic matrix were constructed. A matrix differential calculus was used for this aim. An integral singular equation with respect to an unknown displacement derivative was solved by the method of orthogonal polynomials. The case of steady-state oscillations was investigated. The normal stress on the fixed layer's face was derived. It was analyzed depending on mechanical characteristics of the layer and values of the natural frequencies of vibrations. Formulas for large values of the natural frequencies were constructed for normal stress when both layer's faces were under conditions of smooth contact.

The proposed approach makes it possible to solve elasticity problems with different boundary and initial conditions, so as instead of cylindrical cavity to consider rigid or elastic cylindrical inclusion.

Appendix A. Components of the matrix $\mathbf{\Psi}(\rho)$.

$$\psi_{11} = r_{11} - \frac{s_{11}d_{11} + s_{12}d_{21}}{\bar{\Delta}};\ \psi_{12} = r_{12} - \frac{s_{11}d_{12} + s_{12}d_{22}}{\bar{\Delta}};$$

$$\psi_{21} = r_{21} - \frac{s_{21}d_{11} + s_{22}d_{21}}{\bar{\Delta}};\ \psi_{22} = r_{22} - \frac{s_{21}d_{12} + s_{22}d_{22}}{\bar{\Delta}};$$

$$\bar{\Delta} = \left[\delta_1\delta_2 - \lambda_*^2\right]\cdot\left\{\tfrac{3\kappa-5}{\kappa-1}\lambda_*^2\mathrm{K}_0(\delta_1)\mathrm{K}_1(\delta_2) - \frac{2\lambda_*^2 + p^2}{2\delta_1\delta_2}\left(\tfrac{3\kappa-5}{\kappa-1}\lambda_*^2 + p^2\right)\mathrm{K}_0(\delta_2)\mathrm{K}_1(\delta_1) + \right.$$

$$\left. + \tfrac{3-\kappa}{\kappa-1}\frac{p^2}{\delta_1}\mathrm{K}_1(\delta_2)\mathrm{K}_1(\delta_1) + \tfrac{\kappa+1}{\kappa-1}\lambda_*^2\mathrm{K}_2(\delta_1)\mathrm{K}_1(\delta_2) - \tfrac{1}{2}\tfrac{\kappa+1}{\kappa-1}\frac{\delta_2}{\delta_1}\left(2\lambda_*^2 + p^2\right)\mathrm{K}_2(\delta_2)\mathrm{K}_1(\delta_1)\right\}.$$

Elements of the regular matrix $\mathbf{Y}^R_{\lambda p}(\rho) = \frac{1}{p^2}\begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22}\end{pmatrix}$:

$$r_{11} = -\tfrac{\kappa+1}{\kappa-1}\frac{\lambda_*^2}{\delta_1}I_1(\rho\delta_1) + \tfrac{\kappa+1}{\kappa-1}\delta_2 I_1(\delta_2);\ \ r_{12} = \lambda_* I_1(\rho\delta_1) - \lambda_* I_1(\rho\delta_2);$$

$$r_{21} = \tfrac{\kappa+1}{\kappa-1}\lambda_*\left[I_0(\rho\delta_1) - I_0(\rho\delta_2)\right];\qquad r_{22} = -\delta_1 I_0(\rho\delta_1) + \frac{\lambda_*^2}{\delta_2}I_0(\rho\delta_2);$$

Elements of the singular matrix $\mathbf{Y}^S_{\lambda p}(\rho) = \frac{1}{p^2}\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22}\end{pmatrix}$:

$$s_{11} = \tfrac{\kappa+1}{\kappa-1}\left[-\frac{\lambda_*^2}{\delta_1}\mathrm{K}_1(\rho\delta_1) + \delta_2\mathrm{K}_1(\rho\delta_2)\right];\ \ s_{12} = -\lambda_*\mathrm{K}_1(\rho\delta_1) + \lambda_*\mathrm{K}_1(\rho\delta_2);$$

$$s_{21} = \tfrac{\kappa+1}{\kappa-1}\lambda_*\left[-\mathrm{K}_0(\rho\delta_1) + \mathrm{K}_0(\rho\delta_2)\right];\ \ s_{22} = -\delta_1\mathrm{K}_0(\rho\delta_1) + \frac{\lambda_*^2}{\delta_2}\mathrm{K}_0(\rho\delta_2);$$

Elements of the matrix $\mathbf{D} = \left[\mathbf{A}\cdot\mathbf{Y}^S_{\lambda p}(1) + \mathbf{Y}^{S\prime}_{\lambda p}(1)\right]^{-1}\left[\mathbf{A}\cdot\mathbf{Y}^R_{\lambda p}(1) + \mathbf{Y}^{R\prime}_{\lambda p}(1)\right] = \frac{1}{\Delta}\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22}\end{pmatrix}$:

$$d_{11} = a_{22}b_{11} - a_{12}b_{21};\ \ d_{12} = a_{22}b_{12} - a_{12}b_{22};\ d_{21} = -a_{21}b_{11} + a_{11}b_{21};\ d_{22} = -a_{21}b_{12} + a_{11}b_{22};$$

Elements of the matrix $\mathbf{A}\cdot\mathbf{Y}^S_{\lambda p}(1) + \mathbf{Y}^{S\prime}_{\lambda p}(1) = \frac{1}{p^2}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}$:

$$a_{11} = 2\lambda_*^2\mathrm{K}_0(\delta_1) - \left(2\lambda_*^2 + p^2\right)\mathrm{K}_0(\delta_2) + -\frac{2\lambda_*^2}{\delta_1}\mathrm{K}_1(\delta_1) - 2\delta_2\mathrm{K}_1(\delta_2);$$

$$a_{12} = \tfrac{\kappa-1}{\kappa+1}\lambda_*\left[2\delta_1\mathrm{K}_0(\delta_1) - \frac{2\lambda_* + p^2}{\delta_2}\mathrm{K}_0(\delta_2) + 2\mathrm{K}_1(\delta_1) - 2\mathrm{K}_1(\delta_2)\right];$$

$$a_{21} = \tfrac{\kappa+1}{\kappa-1}\lambda_*\left[\frac{2\lambda_*^2 + p^2}{\delta_1}\mathrm{K}_1(\delta_1) - 2\delta_2\mathrm{K}_1(\delta_2)\right];\quad a_{22} = \left(2\lambda_*^2 + p^2\right)\mathrm{K}_1(\delta_1) - 2\lambda_*^2\mathrm{K}_1(\delta_2).$$

Elements of the matrix $\mathbf{A}\cdot\mathbf{Y}_{\lambda p}^{R}(1)+\mathbf{Y}_{\lambda p}^{R\,\prime}(1)=\frac{1}{p^2}\begin{pmatrix} b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix}$:

$$b_{11}=-\tfrac{3\kappa-5}{2(\kappa-1)}\lambda_*^2 I_0(\delta_1)+\tfrac{3\kappa-5}{2(\kappa-1)}\lambda_*^2 I_0(\delta_2)+\tfrac{1}{2}p^2 I_0(\delta_2)+\tfrac{3-\kappa}{\kappa-1}\left[-\frac{\lambda_*^2}{\delta_1}I_1(\delta_1)+\delta_2 I_1(\delta_2)\right]+$$
$$+\tfrac{1}{2}\tfrac{\kappa+1}{\kappa-1}\left[-\lambda_*^2 I_2(\delta_1)+\left(\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2\right)I_2(\delta_2)\right];$$
$$b_{12}=\tfrac{3\kappa-5}{2(\kappa+1)}\lambda_*\delta_1 I_0(\delta_1)-\frac{\lambda_*}{\delta_2}I_0(\delta_2)\left[\tfrac{3\kappa-5}{2(\kappa+1)}\lambda_*^2+\tfrac{1}{2}\tfrac{\kappa-1}{\kappa+1}p^2\right]+\tfrac{3-\kappa}{\kappa+1}\lambda_*\left[I_1(\delta_1)-I_1(\delta_2)\right]+$$
$$+\tfrac{1}{2}\lambda_*\left[\delta_1 I_2(\delta_1)-\delta_2 I_2(\delta_2)\right];$$
$$b_{21}=\tfrac{\kappa+1}{\kappa-1}\lambda_*\left[\frac{2\lambda_*^2+p^2}{\delta_1}I_1(\delta_1)-2\delta_2 I_1(\delta_2)\right];\quad b_{22}=-\left(2\lambda_*^2+p^2\right)I_1(\delta_1)+2\lambda_*^2 I_1(\delta_2).$$

Appendix B. The form of functions $g_{11}^2(\rho,r),\ g_{11}^1(\rho,r),\ g_{21}^2(\rho,r),\ g_{21}^1(\rho,r)$.

$$g_{11}^1(\rho,r)=\frac{2}{\Delta_{c(r)}}\left[\psi_{11}(\rho)\cdot c_{11}(r)+\psi_{12}(\rho)c_{21}(r)\right];$$
$$g_{11}^2(\rho,r)=\frac{2}{\Delta_{c(r)}\cdot\Delta}\Big[\left\{\psi_{11}(r)\cdot c_{11}(r)+\psi_{12}(r)c_{21}(r)\right\}\cdot\left\{s_{11}(\rho)\cdot r_{22}(r)-s_{12}(\rho)\cdot r_{21}(r)\right\}+$$
$$+\left\{\psi_{21}(r)\cdot c_{11}(r)+\psi_{22}(r)\cdot c_{21}(r)\right\}\cdot\left\{s_{12}(\rho)\cdot r_{11}(r)-s_{11}(\rho)\cdot r_{12}(r)\right\}\Big];$$
$$g_{21}^1(\rho,r)=\frac{2}{\Delta_{c(r)}}\left[\psi_{21}(\rho)\cdot c_{11}(r)+\psi_{22}(\rho)c_{21}(r)\right];$$
$$g_{21}^2(\rho,r)=\frac{2}{\Delta_{c(r)}\cdot\Delta}\Big[\left\{\psi_{11}(r)\cdot c_{11}(r)+\psi_{12}(r)c_{21}(r)\right\}\cdot\left\{s_{21}(\rho)\cdot r_{22}(r)-s_{22}(\rho)\cdot r_{21}(r)\right\}+$$
$$+\left\{\psi_{21}(r)\cdot c_{11}(r)+\psi_{22}(r)\cdot c_{21}(r)\right\}\cdot\left\{s_{22}(\rho)\cdot r_{11}(r)-s_{21}(\rho)\cdot r_{12}(r)\right\}\Big];$$

$$\Delta_{c(r)}=\tilde{r}_{22}\tilde{r}_{11}-\tilde{r}_{12}\tilde{r}_{21}+\frac{1}{\Delta^2}\left[t_{11}t_{22}-t_{12}t_{21}\right]\cdot\left[\tilde{s}_{11}\tilde{s}_{22}-\tilde{s}_{12}\tilde{s}_{21}\right]+$$
$$+\frac{1}{\Delta}\left[\tilde{r}_{21}\left\{\tilde{s}_{11}t_{12}+\tilde{s}_{12}t_{22}\right\}+\tilde{r}_{12}\left\{\tilde{s}_{21}t_{11}+\tilde{s}_{22}t_{21}\right\}-\tilde{r}_{11}\left\{\tilde{s}_{21}t_{12}+\tilde{s}_{22}t_{22}\right\}-\tilde{r}_{22}\left\{\tilde{s}_{11}t_{11}+\tilde{s}_{12}t_{11}\right\}\right];$$
$$\Delta=\tfrac{\kappa+1}{\kappa-1}\frac{\delta_1\delta_2-\lambda_*^2}{\delta_1\delta_2}\left\{\lambda_*^2\mathrm{K}_1(r\delta_1)\mathrm{K}_2(r\delta_2)-\delta_1\delta_2\mathrm{K}_1(r\delta_2)\mathrm{K}_0(r\delta_1)\right\}.$$

Elements of the matrix $\mathbf{Y}_{\lambda p}^{R\,\prime}(r)=\frac{1}{2p^2}\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12}\\ \tilde{r}_{21} & \tilde{r}_{22}\end{pmatrix}$:

$$\tilde{r}_{11}=-\tfrac{\kappa+1}{\kappa-1}\lambda_*^2\left[I_0(r\delta_1)+I_2(r\delta_1)\right]+\tfrac{\kappa+1}{\kappa-1}\left(\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2\right)\left[I_0(r\delta_2)+I_2(r\delta_2)\right];$$
$$\tilde{r}_{12}=\lambda_*\delta_1\left[I_0(r\delta_1)+I_2(r\delta_1)\right]-\lambda_*\delta_2\left[I_0(r\delta_2)+I_2(r\delta_2)\right];$$

$\tilde{r}_{21} = \frac{2(\kappa+1)}{\kappa-1}\lambda_* \left[\delta_1 I_1(r\delta_1) - \delta_2 I_1(r\delta_2)\right]$; $\tilde{r}_{22} = 2\left[-\left(\lambda_*^2 + p^2\right) I_1(r\delta_1) + \lambda_*^2 I_1(r\delta_2)\right]$.

Elements of the matrix $\mathbf{Y}^{S\,\prime}_{\lambda p}(r) = \dfrac{1}{2p^2}\begin{pmatrix} \tilde{s}_{11} & \tilde{s}_{12} \\ \tilde{s}_{21} & \tilde{s}_{22} \end{pmatrix}$:

$$\tilde{s}_{11} = \tfrac{\kappa+1}{\kappa-1}\lambda_*^2\left[\mathrm{K}_0(r\delta_1) + \mathrm{K}_2(r\delta_1)\right] - \tfrac{\kappa+1}{\kappa-1}\left(\lambda_*^2 + \tfrac{\kappa-1}{\kappa+1}p^2\right)\left[\mathrm{K}_0(r\delta_2) + \mathrm{K}_2(r\delta_2)\right];$$

$$\tilde{s}_{12} = \lambda_*\delta_1\left[\mathrm{K}_0(r\delta_1) + \mathrm{K}_2(r\delta_1)\right] - \lambda_*\delta_2\left[\mathrm{K}_0(r\delta_2) + \mathrm{K}_2(r\delta_2)\right];$$

$$\tilde{s}_{21} = \tfrac{2(\kappa+1)}{\kappa-1}\lambda_*\left[\delta_1\mathrm{K}_1(r\delta_1) - \delta_2\mathrm{K}_1(r\delta_2)\right]; \quad \tilde{s}_{22} = 2\left[\left(\lambda_*^2 + p^2\right)\mathrm{K}_1(r\delta_1) - \lambda_*^2\mathrm{K}_1(r\delta_2)\right].$$

Elements of the matrix $\left[\mathbf{Y}^{S}_{\lambda p}(r)\right]^{-1}\cdot\mathbf{Y}^{R}_{\lambda p}(r) = \dfrac{1}{\Delta}\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}$:

$$t_{11} = \tfrac{\kappa+1}{\kappa-1}\left[-\delta_1\mathrm{K}_0(r\delta_1) + \frac{\lambda_*^2}{\delta_2}\mathrm{K}_0(r\delta_2)\right]\cdot\left[-\frac{\lambda_*^2}{\delta_1}I_1(r\delta_1) + \delta_2 I_1(r\delta_2)\right] +$$
$$+ \tfrac{\kappa+1}{\kappa-1}\left[\lambda_*\mathrm{K}_1(r\delta_1) - \lambda_*\mathrm{K}_1(r\delta_2)\right]\cdot\left[\lambda_* I_0(r\delta_1) - \lambda_* I_0(r\delta_2)\right];$$

$$t_{12} = \left[-\delta_1\mathrm{K}_0(r\delta_1) + \frac{\lambda_*}{\delta_2}\mathrm{K}_0(r\delta_2)\right]\cdot\left[\lambda I_1(r\delta_1) - \lambda I_1(r\delta_2)\right] +$$
$$+\left[\lambda\mathrm{K}_1(r\delta_1) - \lambda\mathrm{K}_1(r\delta_2)\right]\cdot\left[-\delta_1 I_0(r\delta_1) + \frac{\lambda_*^2}{\delta_2}I_0(r\delta_2)\right];$$

$$t_{21} = \left(\tfrac{\kappa+1}{\kappa-1}\right)^2\left[\lambda_*\mathrm{K}_0(r\delta_1) - \lambda_*\mathrm{K}_0(r\delta_2)\right]\cdot\left[-\frac{\lambda_*^2}{\delta_1}I_1(r\delta_1) + \delta_2 I_1(r\delta_2)\right] +$$
$$+\left(\tfrac{\kappa+1}{\kappa-1}\right)^2\left[\lambda_* I_0(r\delta_1) - \lambda_* I_0(r\delta_2)\right]\cdot\left[\lambda_*\mathrm{K}_1(r\delta_1) - \lambda_*\mathrm{K}_1(r\delta_2)\right];$$

$$t_{22} = \tfrac{\kappa+1}{\kappa-1}\left[\lambda_*\mathrm{K}_0(r\delta_1) - \lambda_*\mathrm{K}_0(r\delta_2)\right]\cdot\left[\lambda_* I_1(r\delta_1) - \lambda_* I_1(r\delta_2)\right] +$$
$$+ \tfrac{\kappa+1}{\kappa-1}\left[-\frac{\lambda_*^2}{\delta_1}\mathrm{K}_1(r\delta_1) + \delta_2\mathrm{K}_1(r\delta_2)\right]\cdot\left[-\delta_1 I_0(r\delta_1) + \frac{\lambda_*^2}{\delta_2}I_0(r\delta_2)\right].$$

Elements of the matrix $\mathbf{C}(r) = \left[\mathbf{Y}^{S\,\prime}_{\lambda p}(r)\cdot\left[\mathbf{Y}^{S}_{\lambda p}(r)\right]^{-1}\cdot\mathbf{Y}^{R}_{\lambda p}(r) - \mathbf{Y}^{R\,\prime}_{\lambda p}(r)\right]^{-1} = \dfrac{2p^2}{\Delta_{C(r)}}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$:

$$c_{11} = \frac{\tilde{s}_{21}\cdot t_{12} + \tilde{s}_{22}\cdot t_{22}}{\Delta} - \tilde{r}_{22}; \quad c_{12} = -\frac{\tilde{s}_{11}\cdot t_{12} + \tilde{s}_{12}\cdot t_{22}}{\Delta} + \tilde{r}_{12};$$

$$c_{21} = -\frac{\tilde{s}_{21}\cdot t_{11} + \tilde{s}_{22}\cdot t_{21}}{\Delta} + \tilde{r}_{21}; \quad c_{22} = \frac{\tilde{s}_{11}\cdot t_{11} + \tilde{s}_{12}\cdot t_{21}}{\Delta} - \tilde{r}_{11};$$

Appendix C. The form of functions $\psi'_{11}(\rho), \psi'_{12}(\rho), s'_{11}(\rho), s'_{12}(\rho)$

$$\psi'_{11}(\rho) = r'_{11}(\rho) - \frac{s'_{11}(\rho)\cdot d_{11} + s'_{12}(\rho)\cdot d_{21}}{\overline{\Delta}}; \; \psi'_{12}(\rho) = r'_{12}(\rho) - \frac{s'_{11}(\rho)\cdot d_{12} + s'_{12}(\rho)\cdot d_{22}}{\overline{\Delta}};$$

$$r'_{11}(\rho) = -\tfrac{\kappa+1}{\kappa-1}\lambda_*^2\left[I_0\left(\rho\Delta_1\right)+I_2\left(\rho\Delta_1\right)\right]+\tfrac{\kappa+1}{\kappa-1}\left(\lambda_*^2-\tfrac{\kappa-1}{\kappa+1}\omega^2\right)\left[I_0\left(\rho\Delta_2\right)+I_2\left(\rho\Delta_2\right)\right];$$
$$r'_{12}(\rho) = \lambda_*\Delta_1\left[I_0\left(\rho\Delta_1\right)+I_2\left(\rho\Delta_1\right)\right]-\lambda_*\Delta_2\left[I_0\left(\rho\Delta_2\right)+I_2\left(\rho\Delta_2\right)\right];$$
$$s'_{11}(\rho) = \tfrac{\kappa+1}{\kappa-1}\lambda_*^2\left[\mathrm{K}_0\left(\rho\Delta_1\right)+\mathrm{K}_2\left(\rho\Delta_1\right)\right]-\tfrac{\kappa+1}{\kappa-1}\left(\lambda_*^2-\tfrac{\kappa-1}{\kappa+1}\omega^2\right)\left[\mathrm{K}_0\left(\rho\Delta_2\right)+\mathrm{K}_2\left(\rho\Delta_2\right)\right];$$
$$s'_{12}(\rho) = \lambda_*\Delta_1\left[\mathrm{K}_0\left(\rho\Delta_1\right)+\mathrm{K}_2\left(\rho\Delta_1\right)\right]-\lambda_*\Delta_2\left[\mathrm{K}_0\left(\rho\Delta_2\right)+\mathrm{K}_2\left(\rho\Delta_2\right)\right];$$

$$\overline{\Delta} = \left[\Delta_1\Delta_2-\lambda_*^2\right]\cdot\left\{\tfrac{3\kappa-5}{\kappa-1}\lambda_*^2\mathrm{K}_0\left(\Delta_1\right)\mathrm{K}_1\left(\Delta_2\right)-\frac{2\lambda_*^2-\omega^2}{2\Delta_1\Delta_2}\left(\tfrac{3\kappa-5}{\kappa-1}\lambda_*^2-\omega^2\right)\mathrm{K}_0\left(\Delta_2\right)\mathrm{K}_1\left(\Delta_1\right)+\right.$$
$$\left.-\tfrac{3-\kappa}{\kappa-1}\frac{\omega^2}{\delta_1}\mathrm{K}_1\left(\Delta_2\right)\mathrm{K}_1\left(\Delta_1\right)+\tfrac{\kappa+1}{\kappa-1}\lambda_*^2\mathrm{K}_2\left(\Delta_1\right)\mathrm{K}_1\left(\Delta_2\right)-\tfrac{1}{2}\tfrac{\kappa+1}{\kappa-1}\frac{\Delta_2}{\Delta_1}\left(2\lambda_*^2-\omega^2\right)\mathrm{K}_2\left(\Delta_2\right)\mathrm{K}_1\left(\Delta_1\right)\right\}.$$

Appendix D. Deriving formulas for normal stress $\sigma_\xi^0\left(\rho,\xi;\omega\right)$ for large oscillation frequencies

Normal stress in (26) was analyzed in detail. As it is can be seen from expressions for function, which presented in normal stress

$$\Delta_1 = \sqrt{\left(\alpha\pi n\right)^2-\omega^2} = \alpha\pi\sqrt{n^2-\left(\omega/\alpha\pi\right)^2};\quad \Delta_2 = \alpha\pi\sqrt{n^2-\tfrac{\kappa-1}{\kappa+1}\left(\omega/\alpha\pi\right)^2}$$

a negative sign under square root can appear. It depends on value of natural frequencies ω and α which is a ratio of cavity' radius to layer's width. It was identified that necessity of application asymptotic formulas dependent on ratio α. For $\alpha = 1/2$ asymptotic formulas are applied for frequencies starting from $\omega = 2$. For $\alpha = 1/5 -$ from $\omega = 0.7$, for $\alpha = 1/9$ − from $\omega = 0.4$. So, with an increasing of the layer thickness with the same cavity radius the values of frequencies that demand application of asymptotic formulas decrease.

To construct more general formula with no restriction on frequency and value α, the solution to the matrix equation (12) presented with Hankel function was considered again. Taking into account the form function

$$\delta_1 = \sqrt{\lambda_*^2+p^2},\quad \delta_2 = \sqrt{\lambda_*^2+\tfrac{\kappa-1}{\kappa+1}p^2},\quad \lambda_* = \alpha\pi n,\quad \alpha = a/h$$

with Laplace parameter p, three cases of natural number n were considered.

Fist case, when $n > \dfrac{\omega}{\alpha\pi}$.

Formulas that connects Hankel function of imaginary argument with Makdonald function (Gradshtein, Rygik, 1963) were used while translating into the case of steady-state oscillations.

$$H_0^{(1)}(iz) = -\tfrac{2}{\pi}iK_0(z);\; H_1^{(1)}(iz) = -\tfrac{2}{\pi}K_1(z);\; H_2^{(1)}(iz) = \tfrac{2}{\pi}iK_2(z);$$
$$\delta_1 \to \Delta_1 = \sqrt{\left(\alpha\pi n\right)^2-\omega^2};\; \delta_2 \to \Delta_2 = \sqrt{\left(\alpha\pi n\right)^2-\tfrac{\kappa-1}{\kappa+1}\omega^2};$$
$$\Phi_n\left(\rho;\omega\right) = 4\left[\lambda_*^2\Delta_1\Delta_2K_0\left(\rho\Delta_1\right)K_1\left(\Delta_2\right)-\left(\lambda_*^2-\tfrac{1}{2}\omega^2\right)\left(\lambda_*^2+\tfrac{1}{2}\tfrac{3-\kappa}{\kappa+1}\omega^2\right)K_0\left(\rho\Delta_2\right)K_1\left(\Delta_1\right)\right]$$
$$\det{}_n = -4\lambda_*^2\Delta_1\Delta_2K_0\left(\Delta_1\right)K_1\left(\Delta_2\right)+\left(2\lambda_*^2-\omega^2\right)^2K_0\left(\Delta_2\right)K_1\left(\Delta_1\right)-2\omega^2\Delta_2K_1\left(\Delta_2\right)K_1\left(\Delta_1\right);$$

Second case corresponds to a situation, when n belongs to the interval $\sqrt{\frac{\kappa-1}{\kappa+1}}\dfrac{\omega}{\pi\alpha} < n < \dfrac{\omega}{\pi\alpha}$.

In this case definition of Hankel function of first kind was used $H_m^{(1)}(z) = J_m(z) + iN_m(z)$, where $J_m(z)$ - Bessel function, $N_m(z)$ - Neumann function. Instead of function δ_1 function $-i\Delta_3$ was chosen. Real part $\mathrm{Re}\,\sigma_\xi^0(\xi,\rho;\omega)$ was separated to provide reality of the solution while normal stress construction

$$\delta_1 \to -i\Delta_3 = -i\sqrt{\omega^2 - (\alpha\pi n)^2};\ \delta_2 \to \Delta_2 = \sqrt{(\alpha\pi n)^2 - \tfrac{\kappa-1}{\kappa+1}\omega^2};$$

$$\Phi_n(\rho;\omega) = 4[TS + LM]; \qquad \det_n = S^2 - M^2;$$

$$\begin{aligned}
T &= \lambda_*^2\Delta_2\Delta_3 J_0(\rho\Delta_3)K_1(\Delta_2) + \left(\lambda_*^2 - \tfrac{1}{2}\omega^2\right)\left(\lambda_*^2 + \tfrac{1}{2}\tfrac{3-\kappa}{\kappa+1}\omega^2\right)K_0(\rho\Delta_2)J_1(\Delta_3);\\
L &= \lambda_*^2\Delta_2\Delta_3 N_0(\rho\Delta_3)K_1(\Delta_2) + \left(\lambda_*^2 - \tfrac{1}{2}\omega^2\right)\left(\lambda_*^2 + \tfrac{1}{2}\tfrac{3-\kappa}{\kappa+1}\omega^2\right)K_0(\rho\Delta_2)N_1(\Delta_3);\\
S &= 2\omega^2\Delta_2 J_1(\Delta_3)K_1(\Delta_2) - 4\lambda_*^2\Delta_2\Delta_3 J_0(\Delta_3)K_1(\Delta_2) - \left(2\lambda_*^2 - \omega^2\right)^2 J_0(\Delta_3)K_0(\Delta_2);\\
M &= 2\omega^2\Delta_2 N_1(\Delta_3)K_1(\Delta_2) - 4\lambda_*^2\Delta_2\Delta_3 N_0(\Delta_3)K_1(\Delta_2) - \left(2\lambda_*^2 - \omega^2\right)^2 N_0(\Delta_3)K_0(\Delta_2);
\end{aligned}$$

The last situation, when $n < \sqrt{\frac{\kappa-1}{\kappa+1}}\dfrac{\omega}{\alpha\pi}$.

Both function δ_1 and δ_2 were changed by minus imaginary unit by square roots that have positive expressions under them. Here real part of normal stress also has been separated.

$$\delta_1 \to -i\Delta_3 = -i\sqrt{\omega^2 - (\alpha\pi n)^2};\ \delta_2 \to -i\Delta_4 = -i\sqrt{\tfrac{\kappa-1}{\kappa+1}\omega^2 - (\alpha\pi n)^2};$$

$$\Phi_n(\rho;\omega) = -4[CA + DB]; \qquad \det_n = A^2 - B^2;$$

$$\begin{aligned}
C &= \lambda_*^2\Delta_3\Delta_4\left\{J_0(\rho\Delta_3)J_1(\Delta_4) - N_0(\rho\Delta_3)N_1(\Delta_4)\right\} +\\
&\quad + \left(\lambda_*^2 - \tfrac{1}{2}\omega^2\right)\left(\lambda_*^2 + \tfrac{1}{2}\tfrac{3-\kappa}{\kappa+1}\omega^2\right)\cdot\left\{J_0(\rho\Delta_4)J_1(\Delta_3) - N_0(\rho\Delta_4)N_1(\Delta_3)\right\};\\
D &= \lambda_*^2\Delta_3\Delta_4\left\{N_0(\rho\Delta_3)J_1(\Delta_4) - J_0(\rho\Delta_3)N_1(\Delta_4)\right\} +\\
&\quad + \left(\lambda_*^2 - \tfrac{1}{2}\omega^2\right)\left(\lambda_*^2 + \tfrac{1}{2}\tfrac{3-\kappa}{\kappa+1}\omega^2\right)\cdot\left\{N_0(\rho\Delta_4)J_1(\Delta_3) - J_0(\rho\Delta_4)N_1(\Delta_3)\right\};\\
A &= -2\omega^2\Delta_4\left\{J_1(\Delta_3)J_1(\Delta_4) - N_1(\Delta_3)N_1(\Delta_4)\right\} + 4\lambda_*^2\Delta_3\Delta_4\left\{J_0(\Delta_3)J_1(\Delta_4) - N_0(\Delta_3)N_1(\Delta_4)\right\} +\\
&\quad + \left(2\lambda_*^2 - \omega^2\right)^2\left\{J_0(\Delta_4)J_1(\Delta_3) - N_0(\Delta_4)N_1(\Delta_3)\right\};\\
B &= -2\omega^2\Delta_4\left\{N_1(\Delta_3)J_1(\Delta_4) - J_1(\Delta_3)N_1(\Delta_4)\right\} + 4\lambda_*^2\Delta_3\Delta_4\left\{N_0(\Delta_3)J_1(\Delta_4) - J_0(\Delta_3)N_1(\Delta_4)\right\} +\\
&\quad + \left(2\lambda_*^2 - \omega^2\right)^2\left\{N_0(\Delta_4)J_1(\Delta_3) - J_0(\Delta_4)N_1(\Delta_3)\right\};
\end{aligned}$$

Constructed formulas give opportunity to calculate the normal stress for large oscillation frequencies.

References

Arutunyan, N. H., Abramyan, B. L., 1969. Some axisymmetric problems for a half-space and an elastic layer with a vertical cylindrical notch. J. Izv. AN Arm. SSR. Mekhanica, 22 (3), 3−13.

Baoping Hei, Zailin Yang, Yao Wang, Diankui Liu., 2016. Dynamic analysis of elastic waves by an arbitrary cavity in an inhomogeneous medium with density variation. Mathematics and Mechanics of Solids 21 (8), 931−940.

Bardzokas, D. I., Kushnir, D. V., Filshtinskii, L. A., 2009. Dynamic problems of the theory of elasticity for layers and semilayers with cavities. J. Acta Mech. 208, 81−95.

Bobyleva, T., 2016. Approximate method of calculating stresses in layered array. Procedia Engineering 153, 103−106.

Fesenko, A. A., 2019. An exact solution of the dynamical problem for the infinite elastic layer with a cylindrical cavity. Researches in Mathematics and Mechanics. 24, 2 (34), 75–87.

Folias, E. S., Wang, J. J., 1990. On the Three-dimensional stress field around a circular hole in a plate of arbitrary thickness. Comput Mech. 6, 379–391.

Gaoa, M., Zhanga, J. Y., Chenb, Q. S., Gaoc, G. Y., Yangd, J., Lia, D. Y., 2016. An exact solution for three-dimensional (3D) dynamic response of a cylindrical lined tunnel in saturated soil to an internal blast load. Soil Dynamics and Earthquake Engineering 90, 32−37.

Gradshtein, I., Rygik, L., 1963. The tables of integrals, series and products. Moscow, Nauka, pp. 1100.

Grinchenko, V. T., Meleshko, V. V., 1981. Harmonic vibrations and waves in elastic bodies. Kiev, Naukova Dumka, pp. 284.

Grinchenko, V. T., Ulitko, A. F., 1968. An exact solution of the problem of stress distribution near a circular hole in an elastic layer. J. Prikladnaya Mechanica 4 (10), 38–45.

Guz', A. N., 1962. Approximate method for calculation of the stress concentrations around curvilinear holes in shells. Prikl. Mekh. 2 (6), 605–612.

Jain, N. K., Mittal N. D., 2008. Finite element analysis for stress concentration and deflection in isotopic, orthotropic and laminated composite plates with central circular hole under transverse static load. Materials Science and Engineering 498, 115–124.

Kubenko, V. D., 1965. Propagation of elastic waves from a circular hole in an anisotropic inhomogeneous plate. Prikl. Mekh. 1 (2), 25−33.

Linton, C. M., Thompson, I., 2018. Elastic waves trapped above a cylindrical cavity. SIAM J. Appl. Math. 78 (4), 2083−2104.

Malitz, P. Ya., Privarnikov, A. K., 1971. The application of Weber-type transformations to the solution of elasticity problems for layered media with a cylindrical hole. J. Voprosu prochnosty i plastichnossty, 56 – 64.

Menshykov, O., Menshykova, M. & Vaysfeld, N., 2017. Exact analytical solution for a pie-shaped wedge thick plate under oscillating load. Acta Mechanica 228 (12), 4435–4450.

Ming Tao, Ao Ma, Wenzhuo Cao, Xibing Li, Fengqiang Gong., 2017. Dynamic response of pre-stressed rock with a circular cavity subject to transient loading. International Journal of Rock Mechanics and Mining Sciences 99, 1−8.

Muskhelishvili, N. I., 1966. Some basic problems of the mathematical theory of elasticity. Erven P Nordhoff, Ltd., Netherlands, pp. 709.

Novazkiy, W., 1975. The theory of elasticity. Moscow, Mir, pp. 872.

Panasyuk, N. N., 1978. Action of a plane step elastic wave on a spherical cavity. Waves in Continuous Media. Kiev, Naukova Dumka, 79−85.

Popov, G. Ya., 2013. An exact solution of the elasticity theory problem for an infinite layer weakened by a cylindric cavity. Dokladu RUN 451 (5), 1–4.

Popov, G. Ya., Abdimanapov, S. A., Efimov, V. V., 1999. Green's functions and matrix of one-dimensional boundary value problems. Almati, Rauan, pp. 113.

Sneddon, I., 1955. Fourier transforms. Moscow, In. lit., pp. 667.

Timoshenko, S., Goodier, J. N., 1970. Theory of elasticity. McGraw-Hill, Inc., New York, USA, pp. 567.

Uflyand, Ya. S., 1968. Integral transforms in the problems of elasticity theory. Leningrad, Nauka, pp. 402.

Vorovich, I. I., Babeshko, V. A., 1979. Dynamic mixed problems in elasticity theory for nonclassical regions, Nauka, Moscow, pp. 320.

Yahnioglu, N., Babuscu Yesil, U., 2009. Forced vibration of an initial stressed rectangular composite thick plate with a cylindrical hole. ASME International Mechanical Engineering Congress and Exposition IMECE09, Lake BuenaVista, Florida, USA.

Yang, Z., 2009. The stress and strain concentrations of an elliptical hole in an elastic plate of finite thickness subjected to tensile stress. Int J Fract 155, 43−54.

Yih-Hsing Pao., 1983. Elastic waves in solids. J. Appl. Mech. 50 (4), 1152−1164.

Zhou, Y., Zheng, R.-Y., Liu, G.-B., 2011. Dynamic response of elastic layer on transversely isotropic saturated soil to train load. Yantu Lixue/Rock and Soil Mechanics 32 (2), 604−610.

Zhuk, A. P., Kubenko, V. D., Zhuk, Ya. A., 2012. Acoustic radiation force on a spherical particle in a fluid-filled cavity. J. Acoust. Soc. America 132 (4), 2189−2197.